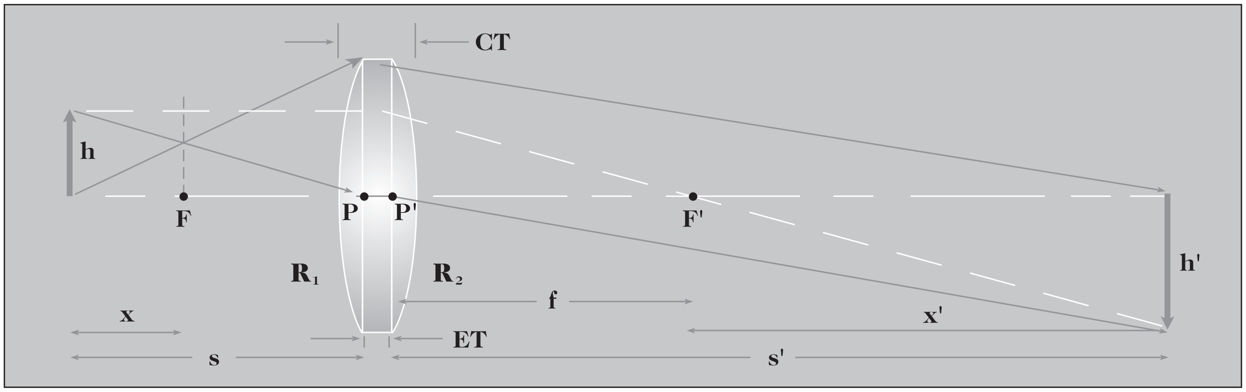
Optical Design Data



LENS FORMULAE

This section gives some basic information regarding the primary lens types and their use in image formation and manipulation. The figure above traces rays of light traveling from object **h** to image **h'** through a lens.

The axial separation of the principal surface, **P**, from its corresponding focal point is the effective focal length, **f.** This is given for a lens with center thickness, **CT**, by the "Thick Lens Formula":

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n-1)CT}{nR_1R_2}\right)$$

where R_1 and R_2 are the radii of curvature and **n** is the index of refraction of the lens material. For a thin lens, **CT** can be assumed to be zero. The effective focal length is then given by:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

The back focal length can be determined by using:

$$BFL = f\left(1 - \frac{(n-1)CT}{nR_1}\right)$$

The Magnification, **m**, of a finite conjugate system is given by:

$$m = \frac{s'}{s} = -\frac{f}{x}$$

The sagittal height of a lens is given by:

$$Sag = |R| - (R^2 - d^2/4)^{\frac{1}{2}}$$

where **R** is the radius of curvature.

The **F**-number of a lens, **F/#**, is the ratio of the focal length, **f**, of a lens system to the diameter, **d**, of its entrance pupil. **F/#** is inversely proportional to twice the Numerical Aperture, NA:

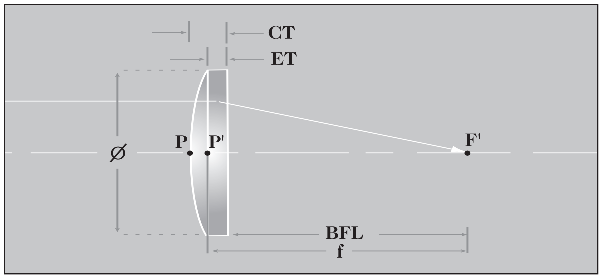
$$F/\# = \left(\frac{f}{d} = \frac{1}{2\,NA}\right)$$

Ignoring the effects of aberrations, the following equations are useful in determining object and image distances and magnification:

- $x = s + f$ $\qquad s' = ms$
- $f = \dfrac{ss'}{s - s'}$ $\qquad xx' = -f^2$
- $x' = s' - f$

LENS TYPES AND SELECTION

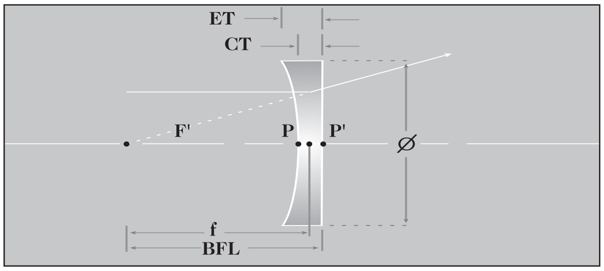
The **plano-convex** lens is the standard focusing optical element. Used with the curved side toward the incident light for focusing, it is useful where optimal spot size is not required. It is most suitable where one conjugate is more than five times the other, such as in sensor applications, or for use with collimated or near collimated light.



The "thick lens formula" from the previous column gives the effective focal length of a **bi-convex** lens:

$$\frac{1}{EFL} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{CT(n-1)}{R_1 R_2 n}\right)$$

Plano-concave lenses are most effective where a negative lens is needed and one conjugate is more than five times the other, e.g. to produce divergent light from a collimated input beam. Having the curved surface facing the collimated beam or the longest conjugate distance produces minimal spherical aberration. For plano-convex and plano-concave lenses, R_2 is infinite ($1/R_2 = 0$).

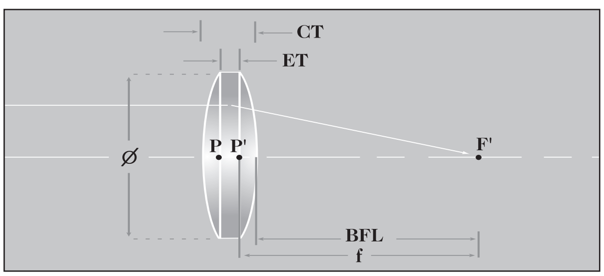


The "thin lens formula" from the previous page as applied to each of these lens types is:

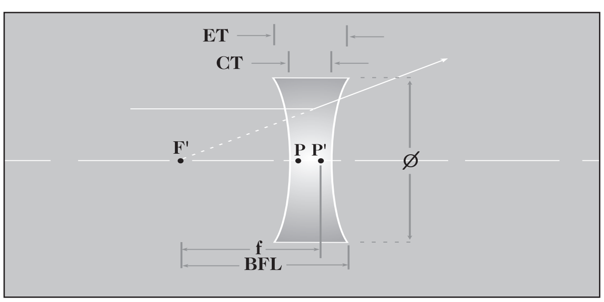
For **plano-convex** and **plano-concave:**

$$\frac{1}{EFL} = \frac{(n-1)}{R_1} \quad \text{or} \quad R_1 = (n-1)(EFL)$$

The **equi-convex** lens is most suitable where the conjugates are on opposite sides of the lenses and the ratio of the distances is less than 5:1, such as in simple image relay components.



Equi-concave lenses are negative lenses and are best for producing diverging light or a virtual image where the input light in converging. For equi-convex and equi-concave lenses, $R_1 = R_2$.

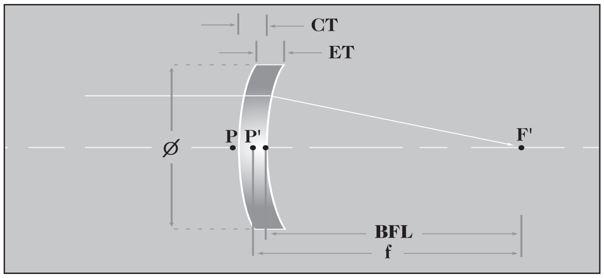


The "thin lens formula" as applied to each of these lens types is:

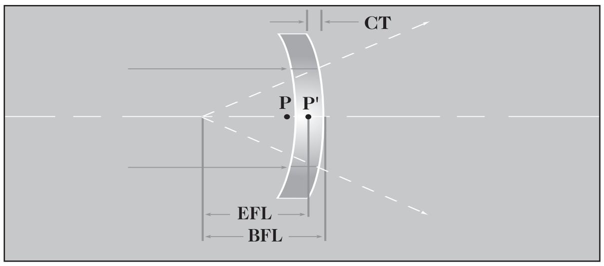
For **equi-convex** and **equi-concave:**

$$\frac{1}{EFL} = \frac{2(n-1)}{R_1} \quad \text{or} \quad R_1 = 2(n-1)(EFL)$$

Positive meniscus lenses may be used to increase the numerical aperture of a positive lens assembly without an undue increase in the aberrations.



The **negative meniscus** lens is best where one conjugate is relatively far from the lens or where both conjugates are on the same side of the lens.



For meniscus lenses, R_1 and R_2 can vary without changing the value of the EFL, so an additional factor is needed along with the EFL in order to uniquely specify an R_1 and R_2. This multiplier (M) is typically related to the EFL and R_1 by:

$$R_1 = (M)(EFL)$$

For either positive or negative meniscus lenses, R_2 may then be found from the Thick Lens Formula. Use a positive value for the EFL of a positive meniscus lens and a negative value for the EFL of a negative meniscus lens:

$$R_2 = (n-1)(EFL)\left(\frac{1 - \frac{(CT)(n-1)}{(M)(EFL)(n)}}{\frac{(n-1)}{M} - 1}\right)$$

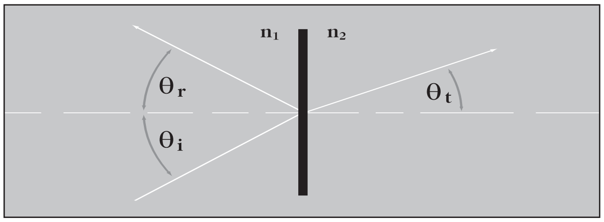
The "thin lens formula" is found by approximating the CT as zero (compared to R_1 and R_2):

$$R_2 = \frac{(EFL)(n-1)}{\frac{n-1}{M} - 1}$$

WINDOWS

A window is used to separate two environments of different pressure, temperature, etc., while allowing light at a specified wavelength to pass between the two. This section briefly discusses the effects of a window on the optical path of light which passes through it. The path taken by each ray through a window is given by Snell's Law:

$$n_i \sin\theta_i = n_t \sin\theta_t$$

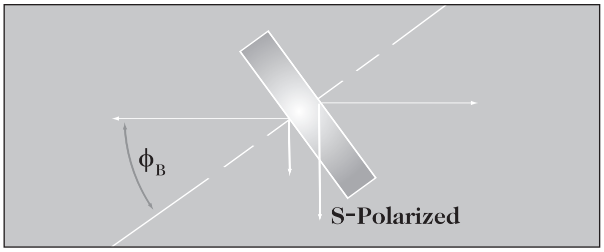


Here n_i is the refractive index of the medium in which the incident light is traveling and n_t is the refractive index of the window material. Usually, there is also a component of light which is reflected from the front surface. The angle the reflected light makes with respect to the normal to the mirror surface is equal to the angle of the incident light with respect to the normal. This is known as the ***Law of Reflection*** and is stated as:

$$\theta_i = \theta_r$$

For a window of any given material, there exists an angle of incidence for which the angle between the reflected and the transmitted light is 90°, as shown below. This is known as the Brewster Angle and is of particular interest because at this angle the reflected light is plane (**s**-) polarized and the refracted light is partially polarized. A stack of windows placed at the Brewster angle will reduce the **s**-polarized component of the incident light until it becomes insignificant. The Brewster Angle is given by:

$$\phi_B = \tan^{-1} n_t/n_i$$

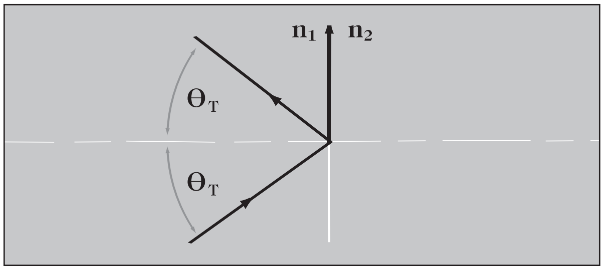


Some of the light traveling within a high refractive index material will reflect off the boundary with a material of lower refractive index. There exists an angle of incidence in this case where all of the light incident on the boundary will be reflected and none is transmitted.

This is known as Total Internal Reflection and the angle at which this occurs is given by:

$$\theta_T = \sin^{-1} n_2/n_1$$

where n_2 is the index of refraction of the "outside" material and n_1 is the index of refraction of the window or higher index material.



CALCULATION OF WINDOW THICKNESS

Minimum thickness of a window required to withstand a pressure difference may be calculated by the following formula:

$$Th = \sqrt{\frac{1.1(P)(DIA)^2}{MR}}$$

Th = thickness, inches
DIA = unsupported diameter, inches
P = pressure difference, psi
MR = modulus of rupture, psi

Pressure at 1 atm = 14.7 psi = 101.324 kPa

Modulus of rupture (MR, psi)
of commonly used crystals:

BaF2	3,900
CaF2	5,300
Ge	10,500
LiF	1,600
MgF2	7,200
Sapphire	65,000
Si	18,100
ZnSe	8,000
ZnS	14,900
ZnS Cleartran	8,700